Algebra 2: Base *e* and Natural Logarithms (ln) Notes

Real Life Application: Suppose a bank compounds interest on accounts ***continuously***, that is, with no waiting time between interest payments.

To develop an equation to determine continuously compounded interest, fill in the table below. Examine what happens to the value of A of an account for increasingly larger numbers of compounding periods, n. (Use the principal P of $1, an interest rate of 100%, and a time t of 1 year.)

n **(frequency of compounding)**	$A = P\left(1+\frac{r}{n}\right)^{nt}$	***A*** **(resulting amount)**
Yearly		
Quarterly		
Monthly		
Daily		
Hourly (8760 hours)		

Base *e* and Natural Logarithms:

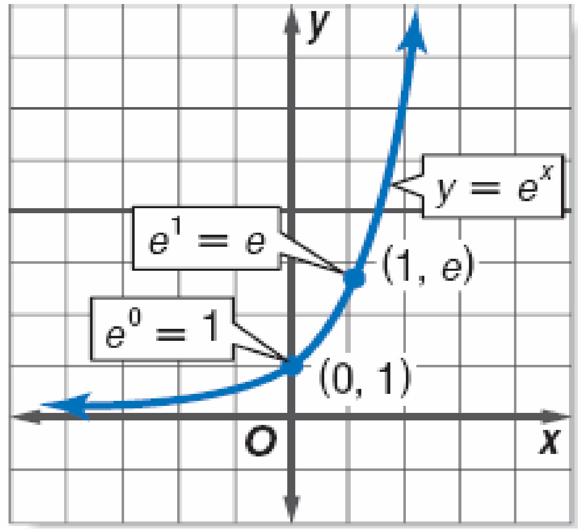
In the table above, as n increases, the expression $1\left(1+\frac{1}{n}\right)^{n(1)}$ or $\left(1+\frac{1}{n}\right)^{n}$ approaches the irrational number 2.71828…. This number is referred to as the **natural base, *e*.**

$$e \approx 2.71828$$

The equation for continuously compounded interest is below:

$$A = Pe^{rt}$$

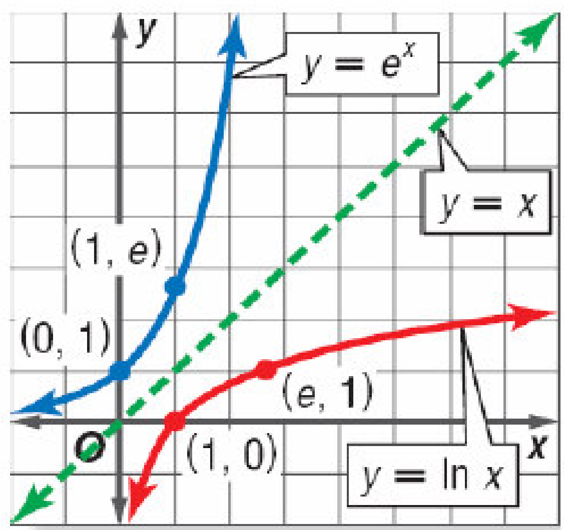
An exponential function with base e is called a **natural base exponential function**. The graph of $y = e^x$ is shown at the right. Natural base exponential functions are used extensively in science to model quantities that grow and decay continuously.



The logarithm with base e is called the **natural logarithm**, sometimes denoted by $\log_e x$, but more often abbreviated ln x.

$$\log_e x = \ln x$$

The **natural logarithmic function**, $y = \ln x$, is the inverse of the natural base exponential function, $y = e^x$. The graphs of these two functions are to the right and show that ln 1 = 0 and ln e = 1.



Use a calculator to evaluate each expression to four decimal places

1. $\ln 732$ **2.** $\ln 84{,}350$ **3.** $\ln 0.735$

Write an equivalent exponential or logarithmic equation.

$e^{15} = x$ $\ln 20 = x$ $e^{3x} = 45$

$\ln (4x) = 9.6$ $e^{8.2} = 10x$ $\ln 0.0002 = x$

$\ln 15 = x$

10.5 Evaluating Expressions with e and ln

Evaluate each expression.

$2^{\log_2 x}$ $\quad$ $8^{\log_8(x^2-3)}$ $\quad$ $n^{\log_n b}$

$\ln e^3$ $\quad$ $e^{\ln 42}$ $\quad$ $e^{\ln 0.5}$ $\quad$ $\ln 0$

$\ln e^{16.2}$ $\quad$ $\ln e^{-1}$ $\quad$ $e^{\ln 3}$ $\quad$ $\ln e^y$

$e^{\ln 2x}$ $\quad$ *ln* -2 $\quad$ *ln* 1 $\quad$ $e^{\ln x^2}$

You can also use *ln* when evaluating using the change of base formula.

Evaluate: $\log_2 5$ can be rewritten as:

or as:

Evaluate the followi

$\log_4 9$ $\quad$ $\log_5 12$

10.5 Solving Equations with e

Example **Solve**

$$\mathbf{3e^{2x} + 2 = 10}$$

$3e^{2x} + 2 = 10$	Original equation
$3e^{2x} = 8$	Subtract 2 from each side.
$e^{2x} = \frac{8}{3}$	Divide each side by 3.
$\ln e^{2x} = \ln \frac{8}{3}$	Property of Equality for Logarithms
$2x = \ln \frac{8}{3}$	Inverse Property of Exponents and Logarithms
$x = \frac{1}{2} \ln \frac{8}{3}$	Multiply each side by $\frac{1}{2}$.
$x \approx 0.4904$	Use a calculator.

Solve $3e^{-2x} + 4 = 10$

$1 - 2e^{2x} = -19$

$\ln 3x = 2$

$\ln (x + 3) = 4$

$\ln x + \ln 2x = 2$

If Sarita deposits $1000 in an account paying 3.4% annual interest compounded continuously, what is the balance in the account after 5 years?

How long will it take the balance in Sarita's account to reach $2000?